POLYMER-FILM COOLING ON A DRUM, TAKING INTO ACCOUNT TEMPERATURE DEPENDENCE OF HEAT CONDUCTION

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UDC 536.21

The cooling of a polymer film is solved by the Biot method, taking into account the temperature dependence of the heat conduction. The solution obtained is numerically verified.

The production of plane films involves the technological operation of cooling a melt on the surface of a thermostatted drum. The effect of the cooling kinetics on the kinetics of the structural changes occurring in the film is significant. The cooling rate is determined by the film thickness and thermal conductivity. This problem may be solved relatively easily for the case of constant thermal conductivity [1]. However, in reality, the process of cooling on the drum occurs over a broad temperature range, i.e., in conditions such that the temperature dependence of the thermal conductivity must be taken into account for the polymers.

This is a problem with internal nonlinearity. Great mathematical difficulty is encountered in solving problems of this type, and so the number of problems which have been analytically solved is very small at present. Such a problem was solved in [2] for a semiinfinite medium.

The aim of the present work is to obtain an analytical solution which is convenient for use in calculations for the problem of film cooling on a drum, taking into account the temperature dependence of the thermal conductivity.

The film thickness is considerably less than the drum diameter, and therefore the drum curvature is neglected. It is also assumed that there is ideal thermal contact between the film surface and the drum.

Suppose that the temperature dependence of the thermal conductivity may be approximated as follows [2]:

$$\lambda = \frac{\lambda_0}{AT^2 + BT \pm 1}, \qquad (1)$$

where λ_0, A, and B are parameters. The expression for the dimensionless temperature is written in the form

$$\theta = \frac{T - T_0}{T_c - T_0}, \qquad (2)$$

where T_0 is the initial temperature of the film; T_c is the drum temperature. In this case, Eq. (1) takes the form

$$\lambda = \frac{\lambda_0}{a\theta^2 + b\theta + d}, \qquad (3)$$

where

$$d = A(T_c - T_0)^2,\ b = (2AT_0 + B)(T_c - T_0),\ d = AT_0^2 + BT_0 \pm 1.$$

The Biot method is used to solve this problem [3]. The problem is formulated as follows: a plate of thickness l, with a thermal conductivity depending on the temperature as in Eq. (3) and a constant volume specific heat ρc, has an initial temperature $\theta = 0$ (Fig. 1). At time t = 0 (contact with the drum), one side of the plate (at x = 0) instantaneously reaches a constant temperature $\theta = 1$; there is no heat transfer at the other side (x = l).

Volgograd Polytechnical Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 38, No. 1, pp. 129-133, January, 1980. Original article submitted April 19, 1979.

0022-0841/80/3801-0091$07.50

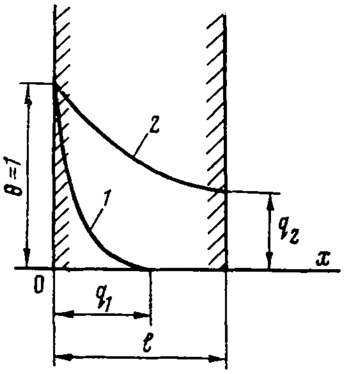

Fig. 1. Dimensionless-temperature distribution in the first (1) and second (2) stages of plate heating. x, m; q_1, m.

The thermal process may be divided into two stages. In the first stage it is assumed that the temperature change occurs over a thickness $x = q_1$ less than the plate thickness l, and the temperature field is satisfactorily approximated as follows:

$$\theta = \left(1 - \frac{x}{q_1}\right)^2. \tag{4}$$

This parabolic approximation is shown in Fig. 1 (curve 1). The heat penetration over the thickness q_1 is a generalized coordinate, defined as a function of the time.

The problem to be solved is one-dimensional, and so it is enough to consider the heat propagation through a cylinder of unit cross-sectional area whose axis is perpendicular to the film surface. The thermal potential is then

$$V_1 = \frac{1}{2}\rho c \int_0^{q_1} \theta^2 dx = 0.1\rho c q_1. \tag{5}$$

The thermal displacement takes the form

$$\rho c \theta = -\frac{dH}{dx}. \tag{6}$$

Taking into account that $H = 0$ at $x = q_1$, it follows that

$$H = \rho c\left(\frac{q_1}{3} - x - \frac{x^2}{q_1} - \frac{x^3}{3q_1^2}\right). \tag{7}$$

The dissipative function takes the form

$$D_1 = \frac{1}{2}\int_0^{q_1} \frac{1}{\lambda} H^2 dx = \dot{q}_1^2 q_1 z \frac{\rho^2 c^2}{2\lambda_0}, \tag{8}$$

where $z = 0.0181818a + 0.025573b + 0.0412698d$.

The generalized thermal force Q_1 is obtained by considering the virtual thermal displacement $\delta H = \rho c \delta q_1/3$ when $x = 0$. The relation

$$Q_1 \delta q_1 = \delta H \tag{9}$$

may be written, and hence

$$Q_1 = \frac{1}{3}\rho c. \tag{10}$$

The Lagrange equation takes the form

$$\frac{\partial V_1}{\partial q_1} + \frac{\partial D_1}{\partial \dot{q}_1} = Q_1. \tag{11}$$

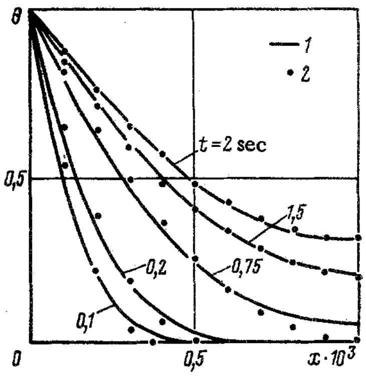

Fig. 2. Calculated temperature distribution in the film at different times: 1) accurate solution; 2) solution by the Biot method. x, m; t, sec.

The solution of Eq. (11) for the initial condition $q_1 = 0$ when $t = 0$ gives the expression

$$q_1 = \sqrt{\frac{7\lambda_0 t}{15\rho cz}}. \tag{12}$$

The first stage of the process ends when q_1 is equal to l, at time t_1:

$$t_1 = \frac{15z\rho cl^2}{7\lambda_0}. \tag{13}$$

In the second stage, corresponding to times $t > t_1$, the dimensionless temperature at the boundary $x = l$ increases. In this stage of the process, the temperature field may also be written as a parabolic dependence,

$$\theta = (1 - q_2)\left(1 - \frac{x}{l}\right)^2 + q_2, \tag{14}$$

which is shown in Fig. 1 (curve 2). The temperature at the boundary $x = l$ is a generalized coordinate. In this case the thermal potential is

$$V_2 = \left(\frac{1}{10} + \frac{2}{15} q_2 + \frac{4}{15} q_2^2\right) \rho cl. \tag{15}$$

The thermal displacement is obtained on integrating Eq. (16) with the temperature value from Eq. (14) and the boundary condition $H = 0$ when $x = l$. The dissipative function is found to be

$$D_2 = \frac{\dot{q}_2^2 \rho^2 c^2 l^3}{2\lambda_0} (\alpha q_2^2 + \beta q_2 + \gamma), \tag{16}$$

where

$$\alpha = 0.060542a;\ \beta = 0.078884a + 0.098765b;$$
$$\gamma = 0.078884a + 0.1171b + 0.215873d.$$

The numerical coefficients in the expressions for z, α, β, and γ are as calculated on a Nairi-S computer.

The generalized force is obtained by considering the virtual thermal displacement $\delta H = (2/3)l\rho c\delta q_2$,

$$Q_2\delta q_2 = \delta H, \tag{17}$$

and hence

$$Q_2 = \frac{2}{3} l\rho c. \tag{18}$$

Substituting Eqs. (15), (16), and (18) into the Lagrange equation

$$\frac{\partial V_2}{\partial q_2} + \frac{\partial D_2}{\partial \dot{q}_2} = Q_2 \tag{19}$$

yields an equation for q_2,

$$\dot{q}_2 = \frac{8\lambda_0}{15\rho c l^2} \cdot \frac{(1 - q_2)}{(\alpha q_2^2 + \beta q_2 + \gamma)}, \tag{20}$$

and integration of this, taking into account that $q_2 = 0$ when $t = t_1$, gives the expression

$$t = t_1 + \frac{15\rho c l^2}{8\lambda_0} \left\{ -\alpha \left[\frac{q_2}{2}(q_2 - 2) + 2q_2 + \ln(1 - q_2) \right] - \beta \left[q_2 + \ln(1 - q_2) \right] - \gamma \ln(1 - q_2) \right\}. \tag{21}$$

In the particular case when A = B = 0, plate cooling occurs in conditions of constant thermal conductivity, and Eqs. (12), (13), and (21) are considerably simplified, coming to correspond to the Biot solution [4]. When A = 0, the temperature dependence of the thermal conductivity is hyperbolic. The Biot method gives an approximate solution of the given nonlinear problem. The error has been estimated by numerical verification, involving the solution of the corresponding boundary-value problem by a difference method [3].

As an example of the calculation, consider the cooling of a polypropylene film for which the variation in thermal conductivity is extremal over the temperature range 100-200°C [5]. The initial data are as follows: volume specific heat $\rho c = 2.23 \cdot 10^6$ J/m$^3 \cdot$°K; T_c = 100°C; T_0 = 200°C; $l = 10^{-3}$ m. The coefficients in Eq. (1) are obtained by least squares, and have the following numerical values: $\lambda_0 = 6.26 \cdot 10^{-2}$ W/m · °K; $A = -5.22 \cdot 10^{-5}$ deg^{-2}; $B = 1.64 \cdot 10^{-2}$ deg^{-1}; the minus sign is taken before the unity in the denominator.

In solving the Fourier equation by the finite-difference method, the grid region had the following characteristics; film-thickness step 10^{-4} m; time interval 10^{-3} sec. Investigating the convergence of the difference scheme allows its error to be estimated; it did not exceed 0.3%. The calculations were performed on a Nairi-S computer.

The results of the numerical verification are shown in Fig. 2. By the Biot method, the first stage is found to end after 0.75 sec. As follows from Fig. 2, the discrepancy of the results does not exceed 6%, i.e., the solution obtained may be used for engineering calculations. Thus, the parabolic approximations of the temperature field in Eqs. (4) and (14) may be used as first approximations. The solution is obtained in the form of algebraic equations, and its numerical realization is considerably simpler than the difference scheme (the machine time is reduced by approximately a factor of 8).

NOTATION

λ	is the thermal conductivity of the material;
λ_0, A, B	are the parameters in the temperature dependence of the thermal conductivity;
T	is the temperature;
a, b, d	are the parameters in the dependence of the thermal conductivity on the dimensionless temperature;
θ	is the dimensionless temperature;
l	is the film thickness;
ρ, c	are the density and specific heat of film material;
t	is the time;
x	is the spatial coordinate;
q_1, q_2	are the generalized coordinates (respectively, the heating depth and the temperature at the boundary);
V	is the thermal potential;
H	is the thermal displacement;
D	is the dissipative function;
z, α, β, γ	are the parameters defined by the thermophysical properties and technological conditions;
Q	is the generalized force;
t_1	is the time of transition from first stage to second.

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RADIANT HEAT TRANSFER IN AN ABSORBENT MEDIUM

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UDC 536.3.33

A dependence is obtained for the radiation flux vector in the form of a series. A calculation formula taking the anisotropy of the radiation field into account is proposed.

Various methods of calculating the radiant heat transfer in an absorbing (radiating) medium are based on a closed and in principle solvable system of differential equations containing the radiant-heat-transfer energy equation and the radiant-transfer equation [1]. For steady heat-transfer conditions, the energy equation may be written in the following form:

$$\operatorname{div} \mathbf{q}_r = 4\sigma (\varepsilon_* T^4 - \alpha T_r^4), \tag{1}$$

where T_r is the radiant temperature, defined at each point of the medium by the expression

$$T_r^4 = \frac{1}{4\sigma} \int_{(4\pi)} I d\omega. \tag{2}$$

The radiation flux vector $\mathbf{q}_r$ may be found by vector integration over the spherical solid angle $\omega = 4\pi$ of the total radiation intensity I, determined from the radiation-transfer equation [1, 2] as follows:

$$I = \frac{\varepsilon_*}{\alpha} B - \frac{1}{\alpha} \frac{dI}{dl}. \tag{3}$$

The equilibrium radiation intensity B at each point of the volume of the medium is then calculated from the well-known formula [2]

$$B = \sigma T^4/\pi. \tag{4}$$

As a rule, the total radiation intensity I is not the same for different directions l, and its dependence on the solid angle ω is not known a priori. Therefore, the integration of Eq. (3) in general form is carried out only for the case of isotropic radiation [2].

It may be shown that on the basis of Eq. (3) calculational dependences may be obtained for the radiation flux vector with an arbitrary configuration of the absorbing-medium volume and an anisotropic radiation field. This involves differentiating repeatedly Eq. (3) with respect to the direction l, taking as constant the ratio between the intrinsic radiation and absorption coefficients of the medium ε_*/α, and neglecting, for simplicity of the equations, the derivatives of second and higher order of the absorption coefficient α. Note that on the right-hand side of each of the resulting equations there is a derivative of the total radiation intensity I of order one higher than that of the derivative of the total radiation intensity on the left-hand side of the equation. Using this structural property of the formulas, the derivative dI/dl may be eliminated from Eq. (3) and the total radiation intensity written as a uniformly converging power series,

$$I = \frac{\varepsilon_*}{\alpha} \left\{ B - \frac{1}{\varkappa_1 \alpha} \frac{dB}{dl} + \frac{1}{\varkappa_1 \varkappa_2 \alpha^2} \frac{d^2 B}{dl^2} + \ldots + \frac{(-1)^n}{\varkappa_1 \varkappa_2 \ldots \varkappa_n \alpha^n} \frac{d^n B}{dl^n} + \ldots \right\}, \tag{5}$$

where $\varkappa_n = (1 + n d\alpha^{-1}/dl)$.

Belgorod Technological Institute of Building Materials. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 38, No. 1, pp. 134-139, January, 1980. Original article submitted March 27, 1979.

0022-0841/80/3801-0095$07.50